Radiative transitions of high energy neutrino in dense matter

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Abstract

The quantum theory of the "spin light" (electromagnetic radiation emitted by a massive neutrino propagating in dense matter due to the weak interaction of a neutrino with background fermions) is developed. In contrast to the Cherenkov radiation, this effect does not disappear even if the medium refractive index is assumed to be equal to unity. The formulas for the transition rate and the total radiation power are obtained. It is found out that radiation of photons is possible only when the sign of the particle helicity is opposite to that of the effective potential describing the interaction of a neutrino (antineutrino) with the background medium. Due to the radiative self-polarization the radiating particle can change its helicity. As a result, the active left-handed polarized neutrino (right-handed polarized antineutrino) converting to the state with inverse helicity can become practically "sterile". Since the sign of the effective potential depends on the neutrino flavor and the matter structure, the "spin light" can change a ratio of active neutrinos of different flavors. In the ultra relativistic approach, the radiated photons averaged energy is equal to one third of the initial neutrino energy, and two thirds of the energy are carried out by the final "sterile" neutrinos. This fact can be important for the understanding of the "dark matter" formation mechanism on the early stages of evolution of the Universe.

A massive neutrino propagating in dense matter can emit electromagnetic radiation due to the weak interaction of a neutrino with background fermions [1, 2]. As a result of the radiation, neutrino can change its helicity due to the radiative self-polarization. In contrast to the Cherenkov

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radiation, this effect does not disappear even if the refractive index of the medium is assumed to be equal to unity. This conclusion is valid for any model of neutrino interactions breaking spatial parity. The phenomenon was called the neutrino “spin light” in analogy with the effect, related with the synchrotron radiation power depending on the electron spin orientation (see [3]).

The properties of the “spin light” were investigated basing upon the quasi-classical theory of radiation and self-polarization of neutral particles [4, 5] with the use of the Bargmann–Michel–Telegdi equation [6] and its generalizations [7, 8]. This theory is valid when the radiated photon energy is small as compared with the neutrino energy, and this narrows the range of astrophysical applications of the obtained formulas.

In the present paper the properties of the “spin light” are investigated basing upon the consistent quantum theory, and this allows the neutrino recoil in the act of radiation to be considered for. The above mentioned restriction is eliminated in this way. On the other hand, the detailed analysis of the results of our investigations shows that the features of the effect depend on the neutrino flavor, helicity and the matter structure. This fact leads to conclusion that the “spin light” can initiate transformation of a neutrino from the active state to practically “sterile” state, and the inverse process is also possible.

When the interaction of a neutrino with the background fermions is considered to be coherent, the propagation of a massive neutrino in the matter is described by the Dirac equation with the effective potential [9, 10]. In what follows, we restrict our consideration to the case of a homogeneous and isotropic medium. Then in the frameworks of the minimally extended standard model, the form of this equation is uniquely determined by the assumptions similar to those adopted in [11]:

$$\left(i\hat{\partial} - \frac{1}{2}\hat{f}(1+\gamma^5) - m_\nu\right)\Psi_\nu = 0. \tag{1}$$

The function f^μ is a linear combination of fermion currents and polarizations, its coefficients can depend on these vectors squared. If the medium is at rest and unpolarized then $\mathbf{f} = 0$. The component f^0 calculated in the first order of the perturbation theory is as follows [12, 13, 14]:

$$f^0 = \sqrt{2}G_{\mathrm{F}}\Big\{\sum_f \left(I_{e\nu} + T_3^{(f)} - 2Q^{(f)}\sin^2\theta_{\mathrm{W}}\right)(n_f - n_{\bar{f}})\Big\}. \tag{2}$$

Here, $n_f, n_{\bar{f}}$ are the number densities of background fermions and anti-fermions, $Q^{(f)}$ is the electric charge of the fermion and $T_3^{(f)}$ is the third component of the weak isospin for the left-chiral projection of it. The parameter $I_{e\nu} = 1$ is equal to unity for the interaction of electron neutrino

with electrons. In other cases $I_{e\nu} = 0$. Summation is performed over all fermions f of the background.

Let us consider the process of emitting photons by a massive neutrino in unpolarized matter at rest. The formula for the spontaneous radiation transition probability of a neutral fermion with anomalous magnetic moment μ_0 is[1]:

$$\begin{gathered} P = -\frac{1}{2p^0}\int d^4x\, d^4y \int \frac{d^4q\, d^4k}{(2\pi)^6}\, \delta(k^2)\delta(q^2 - m_\nu^2)\times \\ \times \mathrm{Sp}\{\varGamma_\mu(x)\varrho_i(x,y;p,\zeta_i)\varGamma_\nu(y)\varrho_f(y,x;q,\zeta_f)\}\varrho^{\mu\nu}_{ph}(x,y;k). \end{gathered} \tag{3}$$

Here, $\varrho_i(x,y;p)$, $\varrho_f(y,x;q)$ are density matrices of initial (i) and final (f) states of the fermion, $\varrho^{\mu\nu}_{ph}(x,y;k)$ is the radiated photon density matrix, $\varGamma^\mu = -\sqrt{4\pi}\mu_0\sigma^{\mu\nu}k_\nu$ is the vertex function. The density matrix of longitudinally polarized neutrino in the unpolarized matter at rest constructed with the use of the solutions of equation (1) has the form

$$\varrho(x,y;p,\zeta) = \frac{1}{2}\Delta^2_{p\zeta}(\hat{p} + m_\nu)(1 - \zeta\gamma^5\hat{S}_p)e^{-i(x^0-y^0)(p^0+f^0/2)+i(\mathbf{x}-\mathbf{y})\mathbf{p}\Delta_{p\zeta}}, \tag{4}$$

where p^μ is the neutrino kinetic moment, $\Delta_{p\zeta} = 1+\zeta f^0/2|\mathbf{p}|$, and $S^\mu_p = \left\{|\mathbf{p}|/m_\nu, p^0\mathbf{p}/|\mathbf{p}|m_\nu\right\}$. Thus, $\zeta = \pm 1$ correspond to the sign of the spin projection on the neutrino kinetic moment.

It is convenient to express the results of calculations using dimensionless variables $\gamma = p^0/m_\nu$, $d = |f^0|/2m_\nu$, $\bar{\zeta}_{i,f} = \zeta_{i,f}\,\mathrm{sign}(f^0)$. The transition rate under investigation is defined as

$$\begin{gathered} W_{\bar{\zeta}_f} = \frac{\mu_0^2 m_\nu^3}{4}\Big\{(1+\bar{\zeta}_f)\left[Z(z_1,1)\Theta(\gamma-\gamma_1) + Z(z_2,-1)\Theta(\gamma-\gamma_2)\right] + \\ +(1-\bar{\zeta}_f)\left[Z(z_1,1)\Theta(\gamma_1-\gamma) + Z(z_2,-1)\Theta(\gamma_2-\gamma)\right]\Theta(\gamma-\gamma_0)\Big\}(1-\bar{\zeta}_i). \end{gathered} \tag{5}$$

Here

$$\begin{aligned} Z(z,\bar{\zeta}_f) = \frac{1}{\gamma(\gamma^2-1)}\Big\{&\ln z\left[\gamma^2 + d\sqrt{\gamma^2-1} + d^2 + 1/2\right] + \\ &+\frac{1}{4}\left(z^2 - z^{-2}\right)\left[d^2\left(2\gamma^2-1\right) + d\sqrt{\gamma^2-1} + 1/2\right] + \\ &+\frac{\bar{\zeta}_f}{4}\left(z - z^{-1}\right)^2\left[2d\sqrt{\gamma^2-1} + 1\right]d\gamma - \\ &-\left(z - z^{-1}\right)\left[d^2 + d\sqrt{\gamma^2-1} + 1\right]\gamma - \\ &-\bar{\zeta}_f\left(z + z^{-1} - 2\right)\left[d\sqrt{\gamma^2-1} + \gamma^2\right]d\Big\}, \end{aligned} \tag{6}$$

[1] In the expression for the radiation energy $\mathcal{E}$, the additional factor k – the energy of radiated photon – appears in the integrand.

where

$$z_1 = \gamma + \sqrt{\gamma^2 - 1} - 2d, \; z_2 = \gamma - \sqrt{\gamma^2 - 1} + 2d,$$

and

$$\begin{aligned} \gamma_0 &= \sqrt{1 + d^2}, \\ \gamma_1 &= \frac{1}{2}\left\{(1 + 2d) + (1 + 2d)^{-1}\right\}, \\ \gamma_2 &= \frac{1}{2}\left\{(1 - 2d) + (1 - 2d)^{-1}\right\}, \quad d < 1/2, \\ \gamma_2 &= \infty, \qquad\qquad\qquad\qquad\qquad\quad d \geqslant 1/2. \end{aligned} \tag{7}$$

Therefore, the transition rate after summation over polarizations of the final neutrino is equal to

$$W_{\bar{\zeta}_f=1} + W_{\bar{\zeta}_f=-1} = \frac{\mu_0^2 m_\nu^3}{2}(1 - \bar{\zeta}_i)\left\{Z(z_1, 1) + Z(z_2, -1)\right\}\Theta(\gamma - \gamma_0). \tag{8}$$

If $d\gamma \ll 1$, then the expression (5) leads to the formula

$$W_{\bar{\zeta}_f} = \frac{16\mu_0^2 m_\nu^3 d^3}{3\gamma}(\gamma^2 - 1)^{3/2}(1 - \bar{\zeta}_i)(1 + \bar{\zeta}_f), \tag{9}$$

obtained in the quasi-classical approximation [2]. In the ultra relativistic limit ($\gamma \gg 1, \; d\gamma \gg 1$), the transition rate is given by the expression

$$W_{\bar{\zeta}_f} = \mu_0^2 m_\nu^3 d^2 \gamma (1 - \bar{\zeta}_i)(1 + \bar{\zeta}_f). \tag{10}$$

Let us consider now the radiation power. If we introduce the function

$$\tilde{Z}(z, \bar{\zeta}_f) = \gamma Z(z, \bar{\zeta}_f) - Y(z, \bar{\zeta}_f), \tag{11}$$

where

$$\begin{aligned} Y(z, \bar{\zeta}_f) = \frac{1}{\gamma(\gamma^2 - 1)} \Big\{ &- \ln z \left[d^2 + d\sqrt{\gamma^2 - 1} + 1\right]\gamma - \\ &- \frac{1}{4}\left(z^2 - z^{-2}\right)\left[d^2 + d\sqrt{\gamma^2 - 1} + 1\right]\gamma + \\ &+ \frac{1}{12}\left(z - z^{-1}\right)^3\left[d^2\left(2\gamma^2 - 1\right) + d\sqrt{\gamma^2 - 1} + 1/2\right] + \\ &+ \frac{1}{2}\left(z - z^{-1}\right)\left[2d^2\gamma^2 + 2d\sqrt{\gamma^2 - 1} + \gamma^2 + 1\right] + \\ &+ \frac{\bar{\zeta}_f}{12}\left(\left(z + z^{-1}\right)^3 - 8\right)\left[2d\sqrt{\gamma^2 - 1} + 1\right] d\gamma - \\ &- \frac{\bar{\zeta}_f}{4}\left(z - z^{-1}\right)^2\left[d\sqrt{\gamma^2 - 1} + \gamma^2\right] d \Big\}, \end{aligned} \tag{12}$$

then the formula for the total radiation power can be obtained from (5), (8) by substitution $Z(z,\bar{\zeta}_f) \to \tilde{Z}(z,\bar{\zeta}_f)$. It can be verified that, if $d\gamma \ll 1$ then the radiation power is

$$I_{\bar{\zeta}_f} = \frac{32\mu_0^2 m_\nu^4 d^4}{3}(\gamma^2-1)^2(1-\bar{\zeta}_i)(1+\bar{\zeta}_f). \tag{13}$$

This result was obtained in the quasi-classical approximation [1]. In the ultra relativistic limit the radiation power is equal to

$$I_{\bar{\zeta}_f} = \frac{1}{3}\mu_0^2 m_\nu^4 d^2\gamma^2(1-\bar{\zeta}_i)(1+\bar{\zeta}_f). \tag{14}$$

It can be seen from equations (10) and (14) that in the ultra relativistic limit the averaged energy of emitted photons is $\langle\varepsilon_\gamma\rangle = \varepsilon_\nu/3$. It should be pointed out that the obtained formulas are valid both for a neutrino and for an anti-neutrino. The charge conjugation operation leads to the change of the sign of the effective potential and the replacement of the left-hand projector by the right-hand one in the equation (1). Thus the sign in front of the γ^5 matrix remains invariant.

The following conclusions can be deduced from the obtained results. A neutrino (anti-neutrino) can emit photons due to coherent interaction with matter only when its helicity has the sign opposite to the sign of the effective potential. Otherwise, radiation transitions are impossible. In the case of low energies of the initial neutrino, only radiation without spin-flip is possible and the probability of the process is very small. At high energies, the main contribution to radiation is given by transitions with the spin-flip, the transitions without spin-flip are either absent or their probability is negligible. This results in the effect of total self-polarization, i. e. the initially left-handed neutrino (right-handed anti-neutrino) are transformed to practically “sterile” right-handed polarized neutrino (left-handed polarized anti-neutrino). For “sterile” particles the situation is opposite. They can be converted to the active form in the medium “transparent” for the active neutrino.

With the use of the effective potential calculated in the first order of the perturbation theory (2), the following conclusions can be made. If the matter consists only of electrons then, in the framework of the minimally extended standard model in the ultra relativistic limit (here we use gaussian units), we have for the transition rate

$$W_{\bar{\zeta}_f} = \frac{\alpha\varepsilon_\nu}{32\hbar}\left(\frac{\mu_0}{\mu_{\mathrm{B}}}\right)^2\left(\frac{\tilde{G}_{\mathrm{F}}\, n_e}{m_e c^2}\right)^2(1-\bar{\zeta}_i)(1+\bar{\zeta}_f), \tag{15}$$

and for the total radiation power

$$I_{\bar{\zeta}_f} = \frac{\alpha\varepsilon_\nu^2}{96\hbar}\left(\frac{\mu_0}{\mu_{\mathrm{B}}}\right)^2\left(\frac{\tilde{G}_{\mathrm{F}}\, n_e}{m_e c^2}\right)^2(1-\bar{\zeta}_i)(1+\bar{\zeta}_f). \tag{16}$$

Here ε_ν is the neutrino energy, $\mu_\mathrm{B} = e/2m$ is the Bohr magneton, α is the fine structure constant, m_e is the electron mass and $\tilde{G}_\mathrm{F} = G_\mathrm{F}(1 + 4\sin^2\theta_\mathrm{W})$, where $G_\mathrm{F},\ \theta_\mathrm{W}$ are the Fermi constant and the Weinberg angle respectively. Thus, after the radiative transition, two thirds of the initial active neutrino energy are carried out by the final "sterile" one.

At the same time, a muon neutrino in the electron medium does not emit any radiation. Moreover, a muon neutrino does not emit radiation in an electrically neutral medium, when the number density of protons is equal to the electron number density. And an electron neutrino can emit radiation if the electron number density is greater than the neutron number density. An example of such medium is the Sun. The neutron medium is "transparent" for all active neutrinos, but an active antineutrino emits radiation in such a medium, the transition rate and the total radiation power can be obtained from equations (15) and (16) after substitution $\tilde{G}_\mathrm{F} \to G_\mathrm{F}$. Therefore the "spin light" can change the ratio of active neutrino of different flavors.

It is obviously that the above conclusions change to opposite if the matter consists of antiparticles. Therefore the neutrino "spin light" can serve as a tool for determination of the type of astrophysical objects, since neutrino radiative transitions in dense matter can result in radiating of photons of super-high energies. This effect can also be important for the understanding of the "dark matter" formation mechanism in early stages of evolution of the Universe.

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